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Persistent currents in a lattice correlated electron ring with a magnetic impurity

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Abstract. The behavior of charge and spin persistent currents in an integrable lattice ring of strongly correlated electrons with a magnetic impurity is exactly studied. Our results manifest that the oscillations of charge and spin persistent currents are similar to the ones, earlier obtained for integrable continuum models with a magnetic impurity. The difference is due to two (instead of one) Fermi velocities of low-lying excitations. The form of oscillations in the ground state is "saw-tooth"-like, generic for any multi-particle coherent one-dimensional models. The integrable magnetic impurity introduces net charge and spin chiralities in the generic integrable lattice system, which determine the initial phase shifts of charge and spin persistent currents. We show that the magnitude of the charge persistent current in the generic Kondo situation does not depend on the parameters of the magnetic impurity, unlike the (magneto)resistivity of transport currents.

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1 Introduction

The Kondo problem [1] describes the effect of a local exchange interaction between the spin of a magnetic impurity and itinerant electrons. For a free electron host spins of conduction electrons screen the spin ($S = \frac{1}{2}$) of the magnetic impurity at low energies, while for large energies the impurity spin remains unscreened. For $S > \frac{1}{2}$ the impurity spin is undercompensated to $S - \frac{1}{2}$ at low energies [2,3]. The crossover energy is referred to as the Kondo temperature. Local moment formation and the screening of the spin is realized within the Anderson's impurity model (see, *e.g.*, [3,4]), where localized electrons of ionic orbitals are hybridized with conduction states. Due to the hybridization the valence of the impurity can range from close to zero (the non-magnetic situation), through the mixed-valence regime to the magnetic or Kondo case (the valence is essentially 1).

Recent several years quantum dots offer the possibility to study the Kondo effect at the level of an *artificial* magnetic impurity [5,6] in a one-dimensional (1D) ring. Unlike the usual impurities in metals [3,4] the study of quantum dots permitted to tune different parameters [7]. Also recently there has been an increased interest in properties of magnetic impurities in correlated electron hosts, where interactions between itinerant electrons affect the behavior of the impurity, see, *e.g.*, [8,9]. Besides purely academic interest, the study of magnetic impurities in correlated electron hosts can be related to experiments on 1D arrays of quantum dots (quantum corrals) [10], and ensembles of magnetic adatoms on metallic surfaces [11]. A number of publications considered persistent currents in quantum rings with magnetic impurities or quantum dots [12–18]. Persistent currents in electron rings are connected with the Aharonov-Bohm-Casher effect [19]. The charge persistent current is the *thermodynamic* characteristic of a ring [20]. It is connected with the Aharonov-Bohm phase shift, which appears when charges move along a loop, pierced by a magnetic flux [19]. Then an external magnetic flux yields nonzero momentum of charges. The charge persistent current is related to the *total orbital moment* of all charges in the ring [20,21]; it is the derivative of the energy of a system in equilibrium with respect to the applied magnetic flux [20,21]. One has to distinguish between persistent currents and transport currents. Recall, transport currents are *kinetic* characteristics of any system [22], characterized by the resistivity and related to it transition amplitude. In the linear response theory the resistivity is the coefficient connecting the value of the current with the value of an applied *electric* field [22]. Hence, the transport current is the consequence of the difference in potentials applied to the source and drain *cf.* Figure 1a.

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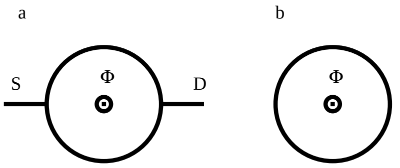

Fig. 1. Different geometries for the manifestation of the Aharonov-Bohm effect of an external magnetic flux Φ in a metallic ring: (a) the transport current geometry with the source (S) and drain (D); (b) the persistent current geometry.

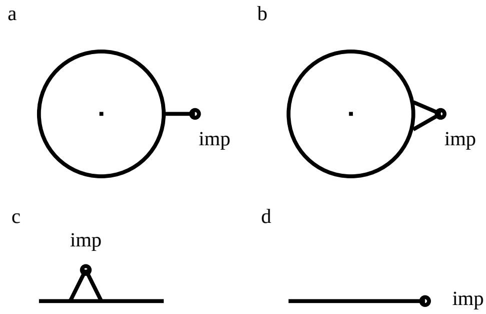

Fig. 2. (a) A side-coupled impurity; (b) an integrable impurity in a ring; (c) an integrable impurity in the bulk of an an open chain; (d) an integrable impurity at the edge of an open chain.

Contrary, the charge persistent current can exist without any applied external electric field: It does not need any source and drain, *cf.* Figure 1b. When the ring between the source and drain is pierced by an external magnetic flux in the geometry of Figure 1a, the transport current is also affected by that flux. Hence, the resistivity of the transport current also becomes flux-dependent. However, such a transport current is *not* exactly equal to the charge persistent current. This difference in the basic natures of transport and persistent currents produces the main difference in the answers, when one considers the effect of a magnetic (Kondo) impurity in a metallic ring, pierced by a flux.

There are two possibilities of coupling of the quantum dot to the ring: embedding of the dot in the ring and side-coupling of the dot. In a recent publication [17] (see, also [23]) it was stated that the geometry of [12,15] pertains to the side-coupled impurity (quantum dot) rather than to embedded one, and that the linearization of the spectrum of host electrons together with the study of only chiral electrons plays the essential role in the properties of charge persistent currents (*cf.* also [18]). The goal of the present study is to find Bethe ansatz results for charge and spin persistent currents in an electron lattice ring with a a magnetic impurity and to compare obtained results with [12–18]. The question to be answered is: What are the consequences of the scaling approximations used in [12–16]? Whether those approximations produce features in the behavior of persistent currents, qualitatively different from the lattice counterpart as [17,18,23] state? To answer this question a convenient choice of the 1D lattice host is the $t-J$ model (Bethe ansatz-solvable at the supersymmetric point, $J = 2t$ [24]). An impurity in such a model reveals non-magnetic, mixed-valence and magnetic (Kondo-like) regimes, depending on the parameters of the model, similar to the Anderson impurity model. Low-lying excitations of that model are also similar to the ones of the Anderson impurity model *cf.* [3,24]. Our study also has to clarify another important issue. There are two independent well-known Bethe ansatz solutions to the Kondo problem: [25] and [26]. These solutions, being different in some details, produce correct answers for thermodynamic characteristics of the magnetic impurity. However, those details produce the difference in the behavior of finite-size corrections (which namely determine persistent currents in metallic systems) [21]. One of those details is the phase factor for the Bethe ansatz equations which govern the behavior of charge degrees of freedom in [25] and the absence of such a phase in [26]. The presence or absence of those phase factors are determined by the particular chosen schemes of taking the scaling limit in these two approaches. In [12] we used the scaling scheme introduced in [25]. This scheme determined the onset of the initial phase shift for charge persistent currents, caused by the Kondo impurity. On the other hand, in [15] we studied persistent currents for a system with multi-channel Kondo impurities, and used the scaling scheme introduced in [26]. Therefore there was no initial phase shift for charge persistent currents (even for the case of the number of channels being equal to 1) in [15]. Hence, the other goal of the present paper is to clarify the situation for initial phase shifts of persistent currents for Bethe ansatz solvable lattice rings with magnetic impurities.

2 Bethe ansatz solution

References [17,23] claimed that the Bethe ansatz approaches of [12–15,18] considered *side-coupled* quantum dots, see Figure 2a. The Bethe ansatz method can be used only for systems with periodic or open boundary conditions. The Bethe ansatz method is properly justified for discrete coordinates of particles because of its main property: Any multi-particle scattering process is considered as a sequence of pair scattering processes between particles in the Bethe ansatz scheme [27]. Thus, when studying continuous limit, one has to use some scaling approximation (regularization procedure) from the lattice counterpart. Therefore the consideration of the continuous scaling limit for the solution of the Kondo problem has some freedom in the determination of the phase shift. Impurities can be included into the lattice Bethe ansatz scheme either as shown in Figures 2b and c for the impurity in the bulk of a ring or an open chain, *i.e.* the impurity is connected with *two* neighboring sites of the host, or connected with only one neighboring site only at the edge of an open chain, as shown in Figure 2d. This is the direct consequence of the fact that impurities can be introduced into the Bethe ansatz monodromies either as special *scattering* matrices (this way implies no reflection off such an impurity at all) or as *boundary reflectors*, which can be applied *only* to the edges of an open chain. Pay attention that we distinguish the reflection and backward scattering (they coincide only when one considers left- and right-movers with dispersion laws, linearized about Fermi points, and neglects states in the "bulk" of the Fermi sea). By the backscattering we mean the transfer from one Fermi point to another. Such processes are present in any lattice Bethe ansatz-solvable theories, like the Heisenberg spin-$\frac{1}{2}$ chain or Hubbard chain [27], but there is no reflection for those models except of at free edges of open chains. Scattering matrices of impurities have to satisfy *Yang-Baxter relations* [27] with scattering matrices of the host, to preserve the integrability. On the other hand, reflectors are described by reflection matrices which satisfy *reflection equations* [28]. However, for any system with open boundary conditions persistent currents are obviously zero. Hence, the only possibility to study persistent currents in Bethe ansatz-solvable models with impurities is to consider impurities, which produce only scattering phases, but not reflections. In papers [12–16], devoted to the influence of magnetic and hybridization impurities on persistent currents, we considered only integrable impurities of this class. The Bethe ansatz solution of the Kondo problem [2,3] also belongs to this class: There Kondo impurities produce only scattering phases, but *not* reflections. The side-coupled impurity (*cf.* Fig. 2a) has, naturally, the properties of a reflector, and, therefore, cannot be introduced into the Bethe ansatz solvable ring *in principle*, because it violates the Yang-Baxter relations. This is why, the claim that the side-coupled impurity could be introduced into the Bethe ansatz-solvable ring is incorrect.

In our study we use the nested Bethe ansatz approach (see, *e.g.*, [29]). The number of operators in the monodromy is given by the number of sites plus impurity [9]. In this approach one can start from the R-matrix [29], which depends on the spectral parameter u. These R-matrices satisfy the standard Yang-Baxter relations $R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u)$. One can introduce usual L-operators for each site of the inhomogeneous lattice [29]. L-operators (including that of the impurity) also satisfy the Yang-Baxter relations $R_{12}(u_1-u_2)L_1(u_1)L_2(u_2) = L_2(u_2)L_1(u_1)R_{12}(u_1-u_2)$ (here the index j denotes the quantum space pertaining to the Hilbert space of the j-th site of the chain and the u_j, $j = 1, ..., L$, are the rapidities of the inhomogeneous lattice). Rapidities parametrize all eigenstates of the Schrödinger equation. The monodromies of the inhomogeneous chain for periodic boundary conditions $\hat{L}^{PBC}$ satisfy the Yang-Baxter relation. The transfer matrices, defined as the traces over the auxiliary spaces of monodromies, mutually commute for different spectral parameters. It constitutes the exact integrability of the problem.

The first derivative of the logarithm of the transfer matrix with respect to the spectral parameter is usually considered as the Hamiltonian. The generic form of the Hamiltonian of the supersymmetric $t-J$ model with the impurity has been derived in [9]. The Hamiltonian consists of two parts, the host Hamiltonian, $\mathcal{H}_{host}$, and the impurity Hamiltonian, $\mathcal{H}_{imp}$. The host Hamiltonian is $\mathcal{H}_{host} = \sum_j \mathcal{H}_{j,j+1}$, where

$$\begin{aligned}\mathcal{H}_{j,j+1} = &-\sum_{\sigma} \mathcal{P}(c^\dagger_{j,\sigma}c_{j+1,\sigma} + c^\dagger_{j+1,\sigma}c_{j,\sigma})\mathcal{P} \\ &+ c^\dagger_{j,\downarrow}c_{j,\uparrow}c^\dagger_{j+1,\uparrow}c_{j+1,\downarrow} + c^\dagger_{j,\uparrow}c_{j,\downarrow}c^\dagger_{j+1,\downarrow}c_{j+1,\uparrow} \\ &- n_{j,\uparrow}n_{j+1,\downarrow} - n_{j,\downarrow}n_{j+1,\uparrow}\ , \qquad (1)\end{aligned}$$

$c_{j,\sigma}$, $c^\dagger_{j,\sigma}$ destroy or create an electron at the site j with spin σ, $n_{j,\sigma} = c^\dagger_{j,\sigma}c_{j,\sigma}$ is the number operator of electrons with the spin σ at the site j and $\mathcal{P} = (1-n_{j,-\sigma})(1-n_{j+1,-\sigma})$ is the projection operator which excludes double occupation of each site. The first term determines the hopping between the neighboring sites (with the hopping matrix element equal to 1), while the other terms define the exchange interaction between electrons at neighboring sites with the exchange constant equal to 2. The impurity's part of the Hamiltonian (for the impurity situated between sites m and $m+1$) is

$$\begin{aligned}\mathcal{H}_{imp} = \frac{(M,\sigma|M+\sigma)}{\theta^2 + (S+\frac{1}{2})^2}\Big(&\mathcal{H}_{m,imp} + \mathcal{H}_{imp,m+1} \\ &- 2S(S-1)\mathcal{H}_{m,m+1} + \{\mathcal{H}_{m,imp}, \mathcal{H}_{imp,m+1}\} \\ &+ i\theta[\mathcal{H}_{m,imp}, \mathcal{H}_{imp,m+1}]\Big)\ , \qquad (2)\end{aligned}$$

where $\{.,.\}$ ($[.,.]$) denote anticommutator (commutator) and $(M,\sigma|M+\sigma)$ denotes the Clebsch-Gordan coefficient $(\frac{1}{2}\sigma, S'M|\frac{1}{2}S'SM+\sigma)$ with $S = S'+\frac{1}{2}$. An integrable impurity embedded in a host lattice is located on a link of the chain and interacts with electrons on both sites joined by the link (*cf.* Figs. 2b and c). All the coupling constants of the impurity Hamiltonian depend on two parameters: S, determining the spin of the impurity, and the off-resonance shift θ (here we limit ourselves with real θ), determining the impurity-host coupling even for $S = \frac{1}{2}$. From equation (2) it is clear, that an impurity of spin $S = \frac{1}{2}$ and $\theta = 0$ is the adding of one more site to the host. On the other hand, the case $\theta \to \infty$ defines the magnetic impurity, totally decoupled from the host ring. The three-site terms of the impurity Hamiltonian violate the T and P symmetries separately, while their product PT is of course invariant [9]. Namely the violation of these symmetries by the Bethe ansatz-solvable magnetic impurity is the reason for possible renormalizations of the initial phase shifts of topological persistent currents in the ring. These terms are total time derivatives in the classical sense and are only important in quantum mechanical aspects [9]. Although the reflection amplitude is zero as a consequence of the integrability, the impurity interacts with both partial waves (forward and backward moving electrons). The three-site terms can be avoided [9] by placing the impurity site at the open end of the host chain, *cf.* Figure 2d. This considerably simplifies the impurity Hamiltonian, since one of the neighboring host sites is absent. However, in such a case persistent currents are zero, because external electromagnetic fluxes can be totally removed from the model with the help of a gauge transformation (open chain). We propose to check our results experimentally. It seems that the experimental realization of the geometry of our Hamiltonian (Fig. 2b) is not more difficult than the one of Figure 2a, when producing quantum dots in quantum rings. The advantage of such a fabrication will be the possibility to compare experimental data with rigorous theoretical results.

The derivation of the Bethe equations is standard and we present it in the Appendix. We point out that we introduce the external magnetic flux Φ and electric flux F [19] ($F = 4\pi\tau$ is the electric flux generated by a string passing through the center of the ring with linear charge density τ, $F_0 = hc/\mu_B$ is the unit electric flux, μ_B is the Bohr magneton) as usual Peierls factors, which are transformed [21] into twisted boundary conditions. The fluxes give rise to charge and spin persistent currents of the Aharonov-Bohm-Casher type in a closed ring configuration. Quantum numbers (rapidities), which parametrize all eigenvalues and eigenfunctions of the Schrödinger equation are determined from the Bethe equations. For twisted boundary conditions our Hamiltonian is diagonalized by the solution of the following Bethe equations

$$\frac{v_j - \theta + \mathrm{i}(2S'+1)/2}{v_j - \theta - \mathrm{i}(2S'+1)/2}\left[\frac{v_j + \mathrm{i}/2}{v_j - \mathrm{i}/2}\right]^L \mathrm{e}^{-\mathrm{i}2\pi\left(\frac{\Phi}{\Phi_0}+\frac{F}{F_0}\right)} = \prod_{\alpha=1}^{M} \frac{v_j - \Lambda_\alpha + \mathrm{i}/2}{v_j - \Lambda_\alpha - \mathrm{i}/2}\ ,\ j = 1, ..., N\ ,$$

$$\frac{\Lambda_\alpha - \theta + \mathrm{i}S'}{\Lambda_\alpha - \theta - \mathrm{i}S'} \prod_{j=1}^{N} \frac{\Lambda_\alpha - v_j + \mathrm{i}/2}{\Lambda_\alpha - v_j - \mathrm{i}/2} \mathrm{e}^{-\mathrm{i}2\pi\frac{2F}{F_0}} = -\prod_{\beta=1}^{M} \frac{\Lambda_\alpha - \Lambda_\beta + \mathrm{i}}{\Lambda_\alpha - \Lambda_\beta - \mathrm{i}}\ ,\ \alpha = 1, ..., M\ , \quad (3)$$

where M is the number of down-spin electrons and N is the total number of electrons in the chain. The eigenfunctions and eigenvalues of the total Hamiltonian are parametrized by the charge rapidities v_j, $j = 1, ...N$, and the spin rapidities, Λ_α, $\alpha = 1, ...M$. The energy of the system is given by

$$E = 2\sum_{j=1}^{N} \frac{1 - 4v_j^2}{1 + 4v_j^2}. \quad (4)$$

The ground state of the one-dimensional correlated electron system is characterized by $N - 2M$ unbound electron states (with real charge rapidities v_j) and M singlet Cooper-like pairs (bound states of electrons with zero total spin) for which the charge rapidities are complex conjugated pairs [24]. The Bethe equations for the ground state can be re-written in terms of real rapidities v_j (unbound electrons, carrying a spin $\frac{1}{2}$ and charge $-e$) and rapidities Λ_α, now characterizing the singlet Cooper-pair-like bound states, which carry zero spin and charge $-2e$. Taking the logarithm of the transformed Bethe equations we get

$$\Theta\left[\Lambda_\alpha, 1\right] + \frac{1}{L}\Theta\left[\Lambda_\alpha - \theta, \left(S + \frac{1}{2}\right)\right] = \frac{2\pi}{L}\left(J_\alpha + \frac{2\Phi}{\Phi_0}\right) + \frac{1}{L}\sum_{j=1}^{N-2M} \Theta\left[\Lambda_\alpha - v_j, 1/2\right] + \frac{1}{L}\sum_{\beta=1}^{M} \Theta\left[\Lambda_\alpha - \Lambda_\beta, 1\right]\ ,$$

$$\Theta\left[v_j, 1/2\right] + \frac{1}{L}\Theta\left[v_j - \theta, S\right] = \frac{2\pi}{L}\left(I_j + \frac{\Phi}{\Phi_0} + \frac{F}{F_0}\right) + \frac{1}{L}\sum_{\alpha=1}^{M} \Theta\left[v_j - \Lambda_\alpha, 1/2\right], \quad (5)$$

where $j = 1, ..., N - 2M$, $\alpha = 1, ..., M$, $\Theta[v, x] = 2\tan^{-1}(xv)$, and the quantum numbers I_j and J_α appear because the logarithm is a multivalued function. The energy is given by

$$E = 2\sum_{j=1}^{N-2M} \frac{1 - 4v_j^2}{1 + 4v_j^2} - 2\sum_{\alpha=1}^{M} (\Lambda_\alpha^2 + 1)^{-1}\ . \quad (6)$$

The quantum numbers completely determine the Bethe solutions of the ground state and the elementary excitations. For the ground state they are symmetrically distributed with respect to zero. The structure of equations (5) is equivalent to the one of the structure of equations (2, 3) of [13] for the metallic ring with the Anderson impurity, taken in the limit $U \to \infty$. The difference appears to be due to the additional parameter for the latter, the resonance level width, which is taken to be unity in the present model. Equations (3, 5) are periodic in F with the period F_0 and in Φ with the periods Φ_0 and $\Phi_0/2$. Hence, they remain invariant under the replacements $(F/F_0) \to \{\{F/F_0\}\}$, $(\Phi/\Phi_0) \to \{\{\Phi/\Phi_0\}\}$ and $(2\Phi/\Phi_0) \to \{\{2\Phi/\Phi_0\}\}$, where $\{\{x\}\}$ denotes the fractional part of x to the nearest (half)integer (*i.e.* to the nearest I_j and J_α). Spin and charge rapidities parametrize each eigenvalue and eigenfunction of the Schrödinger equation, and, therefore, all characteristics of the model reveal those periodicities also.

Then we can use the fact that the ground state energy of a one-dimensional metallic system (*i.e.* the system which has gapless low-lying excitations: The supersymmetric $t-J$ model belongs to this class [24]) can be presented as the series:

$$\frac{E_0}{L} = E_\infty + \frac{E_1}{L} + \frac{E'}{L\ln L} + \cdots + \frac{E_2}{L^2} + \frac{E''}{L^2\ln L} + \ldots \quad (7)$$

where E_∞ determines thermodynamic properties of the host, E_1 and E' describe the thermodynamic behavior of an impurity (or edges of an open chain), E_2 and E'' describe the behavior of excitations, etc. Notice that logarithmic corrections of order of $(L\ln L)^{-1}$, $(L^2\ln L)^{-1}$ etc. exist for systems with the SU(2) spin symmetry. We shall discuss some of those corrections below.

In the thermodynamic limit, *i.e.* $L, N, M \to \infty$ with N/L, M/L fixed, the Bethe equations for the densities of the charge and spin rapidities are

$$\Theta' [v, 1/2] + \frac{1}{L} X(v) = \int d\Lambda \Theta' [v-\Lambda, 1/2] \sigma(\Lambda) + 2\pi[\rho(v) + \rho_h(v)] ,$$
$$\Theta' [\Lambda, \eta] + \frac{1}{L} Y(\Lambda) = \int dv \Theta' [\Lambda - v, 1/2] \rho(v) + \int dz \Theta' [\Lambda - z, 1] \sigma(z) + 2\pi[\sigma(\Lambda) + \sigma_h(\Lambda)] , \quad (8)$$

where $\rho(v)$, $\rho_h(v)$ are distribution functions (densities) for "quasiparticles" and "quasiholes" of unbound electron excitations, respectively, $\sigma(\Lambda)$ and $\sigma_h(\Lambda)$ are the densities for bound states, the prime denotes derivative with respect to the first argument, and $X(v) = \Theta'[v-\theta, S]$, $Y(\Lambda) = \Theta' \left[\Lambda - \theta, (S + \frac{1}{2})\right]$. The internal energy of the system in the thermodynamic limit is

$$E_\infty + \frac{E_1}{L} = 2 \int \rho(v) \left[\frac{1-4v_j^2}{1+4v_j^2}\right] dv - 2 \int \frac{\sigma(\Lambda)}{\Lambda^2+1} d\Lambda. \quad (9)$$

We can also consider the set of integral equations for the "dressed" (by the interaction) energies of the same low-lying excitations, *i.e.*

$$\Theta' [v, 1/2] - \mu - \frac{H}{2} = \frac{1}{2\pi} \int d\Lambda \Theta' [v-\Lambda, 1/2] \Psi(\Lambda) + \varepsilon(v) ,$$
$$\Theta' [\Lambda, 1] - 2\mu = \frac{1}{2\pi} \int dv \Theta' [\Lambda - v, 1/2] \varepsilon(v) + \frac{1}{2\pi} \int dz \Theta' [\Lambda - z, 1] \Psi(z) + \Psi(\Lambda) , \quad (10)$$

where $\varepsilon(v)$ is the "dressed" energy of unbound electron excitations, while $\Psi(\Lambda)$ is the "dressed" energy of singlet pairs, H is the external magnetic field and μ is the chemical potential. The driving terms in equations (8), *i.e.* the terms that do not explicitly depend on ρ and σ, are either of order 1 or of order $1/L$. The terms of order 1 determine the behavior of the host, while the ones of order $1/L$ drive the impurity. Equations (8) are linear integral equations, such that we may write $\rho = \rho^{host} + (1/L)\rho^{imp}$ and $\sigma = \sigma^{host} + (1/L)\sigma^{imp}$, etc., and obtain separate integral equations for the rapidity densities for the host and the impurity. The parameter $\exp(-\pi|\theta|)$ plays the role of the Kondo temperature, T_K, in this model.

3 Features of persistent currents

One can clearly see that equations (8, 9, 10) do not depend on Φ and F explicitly [30]. It respects the fact that neither E_∞, nor E_1 and E' depend on external electro-magnetic fluxes and the Aharonov-Bohm-Casher quantum topological effects reveal themselves in the (highest) corrections of order of L^{-2} etc. The calculations of those corrections can be performed in the framework of the method, developed in [31], based on the use of the Euler-MacLaurin formula. The finite-size (mesoscopic) correction to the energy, E_2, is determined from the formula

$$E_2 = -\frac{\pi}{6}(v_\rho + v_\sigma) + 2\pi v_\rho(\Delta_\rho^+ + \Delta_\rho^-) + 2\pi v_\sigma(\Delta_\sigma^+ + \Delta_\sigma^-) , \quad (11)$$

where the Fermi velocities for low-lying excitations are, respectively,

$$v_\rho = \frac{\partial \varepsilon}{\partial v}(2\pi\rho)^{-1}|_{v=v_0} , \ v_\sigma = \frac{\partial \Psi}{\partial \Lambda}(2\pi\sigma)^{-1}|_{\Lambda=\Lambda_0} , \quad (12)$$

where v_0 is the Fermi point for unbound electron states, defined by $\varepsilon(\pm v_0) = 0$, and Λ_0 is the Fermi point for spin-singlet pairs, defined by $\Psi(\pm\Lambda_0) = 0$. The conformal dimensions $\Delta_i^\pm$ of primary operators are (*cf.* [32] for the homogeneous supersymmetric t–J model without external fluxes)

$$2\Delta_i^\pm = 2n_i^\pm + \left[\frac{z_{\rho,i}(\Delta N_\sigma - \delta_\sigma) - z_{\sigma,i}(\Delta N_\rho - \delta_\rho)}{\det \hat{z}} \pm [z_{i,\sigma}^T(\Delta D_\sigma + \{\{2\Phi/\Phi_0\}\} - d_\sigma) + z_{i,\rho}^T(\Delta D_\rho + \{\{\Phi/\Phi_0\}\} + \{\{F/F_0\}\} - d_\rho)]\right]^2 , \quad (13)$$

where $i = \rho, \sigma$ and $n_i^\pm$ are the numbers of particle-hole excitations at the right and left Fermi points of each of the Dirac seas. Here integers/half-integers ΔN_i denote the changes in the numbers of quasiparticles in the Dirac seas and integers/half-integers ΔD_i represent the numbers of backscattering excitations (particle transfer from the left to the right Fermi points). Both of them are related to the maximal and minimal values of the quantum numbers I_j and J_α.

The dressed charge matrix $\hat{z} = \hat{\xi}|_{v=v_0,\Lambda=\Lambda_0}$ measures correlations between the bands. The elements of $\hat{\xi}$ satisfy the equations

$$1 = \frac{1}{2\pi} \int d\Lambda \Theta' [v-\Lambda, 1/2] \xi_{\rho,\sigma}(\Lambda) + \xi_{\rho,\rho} ,$$
$$0 = \frac{1}{2\pi} \int d\Lambda \Theta' [v-\Lambda, 1/2] \xi_{\sigma,\sigma}(\Lambda) + \xi_{\sigma,\rho} ,$$
$$1 = \frac{1}{2\pi} \int dv \Theta' [\Lambda - v, 1/2] \xi_{\sigma,\rho} + \frac{1}{2\pi} \int dz \Theta' [\Lambda - z, 1] \xi_{\sigma,\sigma}(z) + \xi_{\sigma,\sigma}(\Lambda) ,$$
$$0 = \frac{1}{2\pi} \int dv \Theta' [\Lambda - v, 1/2] \xi_{\rho,\rho} + \frac{1}{2\pi} \int dz \Theta' [\Lambda - z, 1] \xi_{\rho,\sigma}(z) + \xi_{\rho,\sigma}(\Lambda) . \quad (14)$$

The quantities δ_i and d_i renormalize ΔN_i and ΔD_i because of the impurity (and free edges of an open chain).

In the case of twisted boundary conditions external topological fluxes through the ring also contribute to these quantities. The impurity contributions are related to the valence and magnetization of the impurity *via* the Friedel's sum rule [9]. Obviously, neither Fermi velocities, nor the matrix of dressed charges depend on the boundary conditions. The impurity's contributions δ_i and d_i are determined as follows

$$\begin{aligned}
\delta_\rho &= \int_{-v_0}^{v_0} \mathrm{d}v \rho_h^{imp}(v) \ , \ \delta_\sigma = \int_{-\Lambda_0}^{\Lambda_0} \mathrm{d}\Lambda \sigma_h^{imp}(\Lambda) \ , \\
d_\rho &= -\frac{1}{2}\left(\int_{v_0}^{\infty} \mathrm{d}v \rho_h^{imp} - \int_{-\infty}^{-v_0} \mathrm{d}v \rho_h^{imp}\right) \\
&\quad + \frac{1}{4\pi}[x_\rho(\infty) + x_\rho(-\infty)] \ , \\
d_\sigma &= -\frac{1}{2}\left(\int_{\Lambda_0}^{\infty} \mathrm{d}\Lambda \sigma_h^{imp} - \int_{-\infty}^{-\Lambda_0} \mathrm{d}\Lambda \sigma_h^{imp}\right) \\
&\quad + \frac{1}{4\pi}[x_\sigma(\infty) + x_\sigma(-\infty)] \ , \qquad (15)
\end{aligned}$$

where

$$\begin{aligned}
x_\sigma(\Lambda) &= \int_{-v_0}^{v_0} \mathrm{d}v \Theta\left[\Lambda - v, 1/2\right] \rho_h^{imp}(v) \\
&+ \Theta\left[\Lambda - \theta, (S + \frac{1}{2})\right] + \int_{-\Lambda_0}^{\Lambda_0} \mathrm{d}z \Theta'\left[\Lambda - z, 1\right] \sigma_h^{imp}(z) \ , \\
x_\rho(v) &= \Theta\left[v - \theta, S\right] \\
&\quad + \int_{-\Lambda_0}^{\Lambda_0} \mathrm{d}\Lambda \Theta\left[v - \Lambda, 1/2\right] \sigma_h^{imp}(\Lambda) \qquad (16)
\end{aligned}$$

for twisted (periodic) boundary conditions. (Notice that the terms with d_i were missed in one of papers of [9] and in [13].) We point out that d_i and δ_i are determined *modulo* 1, as other phase shifts ΔN_i, ΔD_i and $n_i^{\pm}$.

In the ground state the persistent charge current is $j_c = -L\partial E_0/\partial \Phi$ and the spin persistent current is $j_s = -L\partial E_0/\partial F$. From the above obtained finite size corrections we see that the spin persistent current reveals oscillations, caused by the external electric flux F, with the period of oscillations F_0. The charge persistent current in general situation manifests the interference of oscillations caused by the external magnetic flux Φ with two periods: Φ_0 and $\Phi_0/2$, *cf.* [13]. The oscillations of persistent currents have the "saw-tooth" like form, which is usual for any system with a large number of particles in it.

Neither Fermi velocities of low-lying excitations, v_i, nor "dressed charge" matrix (which determine magnitudes of persistent currents) explicitly depend on the parameters of the magnetic impurity, S and θ. It is in a drastic contrast to the valence, magnetic susceptibility and specific heat of the impurity, or transport characteristics, like the resistivity or magnetoresistivity. Only *initial shifts* of persistent currents are determined by these parameters in the main ($\sim L^{-2}$) effect. (Generally speaking, persistent currents depend on the hybridization of the impurity to the host, which is put equal to 1 in the present model.) Initial phases of oscillations of persistent currents also depend on the parity of the number of electrons N and down-spin electrons M mod (4) *via* $\Delta D_{\rho,\sigma}$ (*cf.* [12–16]). The nonzero θ determines the nonzero ground state momentum of the system (due to topological charge and spin currents, *i.e.* three-site terms, caused by the integrable impurity), which is nothing else than the nonzero charge and spin chirality (or Noether's topological currents). Hence, the magnetic integrable impurity *does* cause the initial phase shift of persistent currents for the lattice electron model. This points out that the regularization scheme of [25] for the continuum (scaling) limit is more close to the situation with the impurity on the lattice from the viewpoint of finite-size corrections, and a generic integrable impurity in a closed ring has to manifest initial phase shifts in the behaviors of charge and spin persistent currents.

The structure of Φ- and F-dependent terms for our lattice model with the magnetic impurity is similar to the ones of continuum models [12–16] (*cf.* especially [13], where persistent currents were studied for the Anderson impurity model). Hence, the division into left- and right movers (present in [12–16]) is *not* essential for the behavior of the oscillations of charge and spin persistent currents in an exactly solvable ring with an impurity, in spite of [17,18]. Notice that when comparing our model with the Anderson impurity model, one has to take the limit $U \to \infty$ in the latter (which is rather standard for the Kondo case [3,4]). Our answer, equation (11), is obtained in the conformal limit, where dispersion laws of low-lying gapless excitations are linearized about their Fermi points ($\pm v_0$ and $\pm\Lambda_0$). However, that linearization was performed *after* we found the structure of the ground state (*i.e.* the Fermi seas for true low-lying excitations), but not *before*, as was done for the continuum models [12–15]. This constitutes the important difference: There exist *two* Fermi velocities for low-lying excitations in the lattice model, while for the continuum models only one velocity exists for both types of low-lying excitations (it is equal to the Fermi velocity of noninteracting electrons). The onset of two Fermi velocities is the "trademark" of the Luttinger liquid.

The next order corrections (of order of $(L^2 \ln L)^{-1}$, etc.) produce the impurity's contribution to the magnitude of the persistent current (*cf.* [17]). However, for the generic situation of long enough systems those contributions can be neglected when comparing with the main effect of order of L^{-2}. Moreover, any, even very small, magnetic anisotropy will remove logarithmic corrections. Probably the effect of the Kondo "screening cloud" $\xi_K \sim \hbar v_F/T_K$ can be manifested for small enough systems (where $\xi_K \sim L$ at least). However at such conditions the Abrikosov-Suhl (Kondo) resonance is shifted from the Fermi point and the impurity rather reveals the mixed-valence properties. Actually this limit ($\xi_K \geq L$) has no other, standard for the Kondo situation, features. For example, it is well known that the generic Kondo effect (at $\xi_K \ll L$) affects the Sommerfeld coefficient of the low-T specific heat and magnetic susceptibility of the magnetic impurity: They become large (actually, they are inverse proportional to the small T_K) [2–4,22]. However, for the case $\xi_K \geq L$ the specific heat and magnetic susceptibility are *exponentially small* at low T [34]. Similar conclusions (*i.e.* that for $\xi_K \geq L$ the specific heat and magnetic susceptibility are exponentially small at low T) can be obtained also for the supersymmetric $t-J$ model with the integrable magnetic impurity. Results for this model for small finite L, N and M are reported elsewhere [35].

For the half-filling (where the valence of the impurity is 1, reminiscent to the standard Kondo situation [3,4]) for $H = 0$ the magnitudes of oscillations of persistent currents with the periods Φ_0 and $\Phi_0/2$ become equal to each other, as well as the initial phases of those oscillations. Hence, in this case the charge persistent current must reveal only one period of oscillations (Φ_0). However, for any nonzero H and non-half filled case the magnitudes of oscillations with periods Φ_0 and $\Phi_0/2$ become different from each other and the interference of those two periodic functions has to manifest itself in the ground state. It turns out that as for any 1D system with the SU(2) symmetry present, small deviations of H from zero produce logarithmic corrections related to marginal corrections in the renormalization group sense. Oscillations of charge and spin persistent currents are maximal in the ground state, their main effect is of order of L^{-2}. However, for $T \neq 0$ persistent currents become exponentially small with L [21]. The "saw-tooth"-like form of oscillations is related to only the ground state. Any nonzero temperature $T > \hbar v_F/L$ strongly reduces the magnitudes of the most of harmonics, and sinusoidal-like oscillations result.

The magnetoresistivity of the considered model for $S = \frac{1}{2}$ in the ground state was calculated in [9] (without external fluxes, but their role is transparent: They shift the phases, which determine the resistivity). At low T the lowest thermal corrections to the resistivity for $S = \frac{1}{2}$ are proportional to T^2, as usual for Fermi liquids. On the other hand, for quantum dots the resonant Kondo transparency (tunneling) in the ground state was first calculated in [36,37] for a simple metal as a host. Actually we can use the result [36,37] $G = \sum_\sigma G_0 \sin^2 \delta_\sigma$ with the phase shifts of the electrons δ_σ determined from the values of δ_i *via* the Friedel sum rule, see [9]. Notice that G_0 can depend on Φ and F as a harmonic function of Φ/Φ_0 and F/F_0, because it is determined by the matrix elements, which connect the dot to the ring and the dependence of the topological fluxes can be transfered to those elements with the help of the gauge transformation. We emphasize that unlike charge persistent currents, the characteristics of transport currents *do* depend on the parameters of the magnetic impurity. It is the direct consequence of the difference in definitions of those two types of currents. The Abrikosov-Suhl (Kondo) resonance (which is determined by the *spin* low-lying excitations) strongly affects the transmition amplitude (and, hence, the resistivity) of the chain, but it does not affect the total orbital moment of the ring (*i.e.* the charge persistent current). The temperature dependence of characteristics of transport currents of the considered model reveals the usual Kondo crossover at T_K.

4 Summary

In conclusion, in this paper we have studied the behavior of charge and spin persistent currents in an integrable lattice ring of strongly correlated electrons with a magnetic impurity. Our results manifest that oscillations of charge and spin persistent currents are similar to the ones, earlier obtained for integrable continuum models with a magnetic impurity. The form of oscillations in the ground state is "saw-tooth"-like, generic for any multi-particle coherent one-dimensional models. For very small sizes of quantum rings those ground state "saw-tooth" oscillations can be replaced by harmonic ones, however, the generic Kondo effect is absent for such small rings in which the size of the ring becomes comparable with the size of the Kondo "screening cloud" ξ_K. The magnitude of oscillations is determined by two Fermi velocities of low-lying spin and charge excitations and by the matrix of "dressed charges", which reveal the interaction between electrons in the system. We have shown that the linearization of the dispersion law of itinerant electrons does not produce any qualitative renormalization of the answers for persistent currents (except of two, instead of one, characteristic Fermi velocities for low-lying excitations, characteristic for any interacting one-dimensional electron system). Our result for charge and spin persistent currents agrees with the ones for the approximation, which considered only electrons close to the right or left Fermi points, but with two Fermi velocities, unlike the only one for the continuous approximation. The integrable magnetic impurity introduces the net charge and spin chiralities in the generic integrable lattice system. Those chiralities determine initial phase shifts of charge and spin persistent currents, and they appeared also in some continuum models. We propose to check our theoretical results in experiments on quantum rings with quantum dots in the geometry, similar to the integrable impurity situation. We argue that the advantage of such measurements will be the possibility to compare the data of experiments with the exact theoretical results. Finally, some remark is in order. Our results are generic only for *integrable* impurities (which, on the other hand, very well describe thermodynamic and kinetic characteristics of magnetic Kondo impurities [2–4]). However, one can only speculate, whether these results are generic for the experimental realizations of artificial magnetic impurities (quantum dots) in quantum rings or not. If they are not, then this will be the restriction for the use of Bethe ansatz-solvable models for the description of quantum dots in quantum rings. We expect that further theoretical and experimental studies of persistent currents in quantum rings with magnetic impurities will clarify most of those problems.

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Appendix

Here we present the proof of the exact integrability of our model and the derivation of the Bethe equations equations (3) following the procedure of [38]. We start from the gl(1|2) invariant R-matrix

$$R^{\alpha\gamma}_{\beta\delta}(u) = a(u)(-1)^{p(\alpha)p(\gamma)}\delta_{\alpha,\beta}\delta_{\gamma,\delta} + [1 - a(u)]\delta_{\alpha,\delta}\delta_{\gamma,\beta} \ , \quad \text{(A.1)}$$

where $\alpha, \beta, \gamma, \delta = 1, 2, 3$, $a(u) = u/(u + i)$, and we choose the grading, *e.g.*, $p(1) = 0$, $p(2) = p(3) = 1$. This R-matrix satisfies the Yang-Baxter relation. The monodromy matrix $\hat{L}^{PBC}(u)$ forms 3×3 matrix with respect to the states of the auxiliary particle. For this grading it is convenient to introduce additional matrix $\check{R}^{\alpha\gamma}_{\beta\delta} = R^{\gamma\alpha}_{\beta\delta}$, with which the monodromies satisfy the Yang-Baxter relation $\check{R}(u - v)(\hat{L}^{PBC}(u) \otimes_s \hat{L}^{PBC}(v)) = (\hat{L}^{PBC}(v) \otimes_s \hat{L}^{PBC}(u))\check{R}(u - v)$, where $\otimes_s$ denotes the super tensor product associated with our choice of the grading. The transfer matrix $t(u)$ is determined as the supertrace of the monodromy $t(u) = \mathrm{str}\hat{L}^{PBC}(u) \equiv (-1)^{p(\alpha)}\hat{L}^{PBC}_{\alpha\alpha}(u)$. As we pointed out above, from the Yang-Baxter relations for the monodromy it follows that $[t(u), t(v)] = 0$, hence, $t(u)$ and $t(v)$ have common eigenfunctions, which means that any functions of the transfer matrix $t(u)$ commute mutually and with the transfer matrix. This implies the infinite set of conservation laws, *i.e.* the exact integrability of the problem. The Hamiltonian of our problem is defined as the logarithmic derivative of the above transfer matrix with respect to the spectral parameter taken at $u = 0$.

Now our task is to find the eigenfunctions and eigenvalues of $t(u)$. We start with the vacuum vector Ω_0, chosen so that $\hat{L}^{PBC}_{\alpha\beta}(u)\Omega_0 = 0$ for $\alpha > \beta$. The action of the diagonal matrix elements is considered as $\hat{L}^{PBC}_{\alpha\alpha}(u)\Omega_0 = a_\alpha(u)\Omega_0$, where $\alpha = 1, 2, 3$. We can consider any eigenstate in the form

$$\Omega(v'_1, \ldots, v'_N) = \prod_{k=1}^{N} \hat{L}^{PBC}_{\alpha_k\beta_k}(v'_k)\Omega_0 \ , \ \alpha_k < \beta_k \ . \quad \text{(A.2)}$$

Following [38] we can show that the action of the transfer matrix $t(u) = \hat{L}^{PBC}_{11}(u) - \hat{L}^{PBC}_{22}(u) - \hat{L}^{PBC}_{33}(u)$ onto the vector equation (A.2) produces the same vector times the eigenvalue

$$\tau(u) = a_1(u)\prod_{j=1}^{N} a^{-1}(v'_j - u) - a_3(u)\prod_{k=1}^{M} a^{-1}(\Lambda_k - u) - a_2(u)\prod_{j=1}^{N} a^{-1}(v'_j - u)\prod_{k=1}^{M} a^{-1}(u - \Lambda_k) \ , \quad \text{(A.3)}$$

where $\{\Lambda_k\}_{k=1}^{M}$ is the additional set of rapidities, [38,29] and so-called "unwanted terms". The conditions of the cancellation of those "unwanted terms" can be written as [38]

$$\frac{a_2(\Lambda_k)}{a_3(\Lambda_k)}\prod_{j=1}^{N} a^{-1}(v'_j - \Lambda_k) = \prod_{\substack{l=1\\ l\neq k}}^{M} \frac{a(\Lambda_k - \Lambda_l)}{a(\Lambda_l - \Lambda_k)} \ ,$$
$$\frac{a_1(v'_j)}{a_2(v'_j)} = \prod_{k=1}^{M} a^{-1}(v'_j - \Lambda_k) \ , \quad \text{(A.4)}$$

During the derivation of equations (A.3) and (A.4) the concrete form of $a_{1,2,3}(u)$ was not used, [38] but only the triangular action of $\hat{L}^{PBC}(u)$ onto the vacuum state and the c-number form of the action of the diagonal matrix elements were supposed.

Consider now the representation of the diagonal matrix elements of $\hat{L}^{PBC}(u)$ for our supersymmetric $t-J$ model with an impurity. Let us consider the unity operator $\sum_j I_j$, the operator of the total number of electrons $\hat{N}$, and three operators of the projections of the total spin of the system, $S^{\pm,z}$, respectively. They form $U(1)$ and $SU(2)$ subalgebras ($[S^z, S^\pm] = \pm S^\pm$, $[S^+, S^-] = 2S^z$) of gl(2|1). The fermion operators $Q^\pm_{1,2}$ satisfy the anticommutation relations (see, *e.g.*, [39])

$$\{Q^\pm_1, Q^\pm_2\} = \pm\frac{S^\pm}{2} \ , \{Q^\pm_1, Q^\mp_2\} = \pm\frac{-S^z \pm \hat{N}}{2}\cdot \quad \text{(A.5)}$$

with other mutual anticommutators being zero. They satisfy the commutation relations with the bosonic generators

$$[S^z, Q^\pm_l] = \pm\frac{Q^\pm_l}{2} \ , \ [\hat{N}, Q^\pm_l] = (-1)^{l+1}\frac{Q^\pm_l}{2} \ ,$$
$$[S^\mp, Q^\pm_l] = Q^\mp_l \ , \ [S^\pm, Q^\pm_l] = 0 \ , \quad \text{(A.6)}$$

with $l = 1, 2$. In the basis, where $\hat{N}$, $\mathbf{S}^2$ and S^z are diagonal, the non-vanishing matrix elements of $Q^\pm_{1,2}$ are

$$\langle S + \frac{1}{2}, S - \frac{1}{2}, \sigma \pm \frac{1}{2}|Q^\pm_1|S, S, \sigma\rangle = \pm\sqrt{\frac{S \mp \sigma}{2}} \ ,$$
$$\langle S, S, \sigma|Q^\pm_2|S + \frac{1}{2}, S - \frac{1}{2}, \sigma \mp \frac{1}{2}\rangle = \sqrt{\frac{S \pm \sigma}{2}}\cdot \quad \text{(A.7)}$$

Actually these operators are the sums of local operators of the same structure at each site of the system. For $S = \frac{1}{2}$ one can express these operators in terms of the standard electron creation and annihilation operators as $\hat{N} = \sum_j(n_{j,\uparrow} + n_{j,\downarrow})$, $2S^z = \sum_j(n_{j,\uparrow} - n_{j,\downarrow})$, $S^\mp = \sum_j c^\dagger_{j,\downarrow,\uparrow}c_{j,\uparrow,\downarrow}$, $Q^+_1 = \sum_j(1 - n_{j,\downarrow})c^\dagger_{j,\uparrow}$, $Q^+_2 = \sum_j(1 - n_{j,\uparrow})c_{j,\downarrow}$, and $Q^-_{1,2} = (Q^+_{1,2})^+$. The multipliers $(1 - n_{j,\sigma})$ of fermionic operators Q exclude double occupations of each site, as it must be for the $t-J$ model.

Let us construct the local L-operators (at site j) of the supersymmetric $t-J$ model in the form $L_j(u) = a(u)\hat{I}^j + [1 - a(u)]\hat{Z}^j$, where $\hat{Z}^j_{12} = (Q^+_2)_j$, $\hat{Z}^j_{13} = (Q^-_1)_j$, $\hat{Z}^j_{21} = (Q^+_1)_j$, $\hat{Z}^j_{31} = (Q^-_2)_j$, $\hat{Z}^j_{23} = S^-_j$, $\hat{Z}^j_{32} = -S^+_j$, $\hat{Z}^j_{11} = [I_j - (\hat{N}_j/2) + S^z_j][I_j - (\hat{N}_j/2) - S^z_j]$, $\hat{Z}^j_{22} = -[I_j - (\hat{N}_j/2) - S^z_j][(\hat{N}_j/2) + S^z_j]$, and

$\hat{Z}^j_{33} = -[I_j - (\hat{N}_j/2) + S^z_j][(\hat{N}_j/2) - S^z_j]$, where $\hat{I}^j$ is the unitary 3×3 matrix. The action of the L-operator on the vacuum state obviously produces c-number diagonal elements and zeros for $[L_j]_{\alpha,\beta}$ for all $\alpha > \beta$. Then we construct the monodromy as $\hat{L}^{PBC}(u) = \hat{T}L^S_{imp}(u - \theta)L^{1/2}_1(u)\cdots L^{1/2}_L(u)$, where for the host sites we choose $S = \frac{1}{2}$, and $\hat{T}$ denote the diagonal matrix, which accounts electro-magnetic fluxes. Naturally, the elements of such a monodromy acting on the vacuum state produce zero, $\hat{L}^{PBC}_{\alpha\beta}(u)\Omega_0 = 0$ for $\alpha > \beta$, while the diagonal matrix elements are c-numbers, which were the only conditions of the above introduced construction. For this representation we have $a_1(u) = 1$, $a_3(u) = \exp(\mathrm{i}\Phi_\downarrow)a^L(u)a_{S'}(u - \theta)$, and $a_2(u) = \exp(\mathrm{i}\Phi_\uparrow)a^L(u)a_{S'}(u-\theta)Z(u-\theta)$, where $a_{S'}(x) = (x + \mathrm{i}S')/[x + \mathrm{i}(S' + 1)]$ (notice that $a(x) \equiv a_{S'=0}(x)$)and $Z(x) = (x-\mathrm{i}S')/(x+\mathrm{i}S')$. Naturally, for $\theta = 0$ and $S' = 0$ ($S = \frac{1}{2}$) the impurity site is just an additional host site. One can see that for this choice of $a_{1,2,3}$ in equations (A.4) one obtains equations (3) for $v_j = v'_j - i/2$. These equations and equation (A.3), naturally, coincide (up to a sign) for $\Phi_{\uparrow,\downarrow} = 0$ with the Bethe ansatz equations and the equation for the eigenvalue of the transfer matrix of [9] (*e.g.*, for $S \to S'$ with equations (A.1, A.2) of the third reference of [9], where these equations were obtained in the framework of the grading of the second reference of [29]). The integrability exists for impurity L-operators with different values of θ, so that a finite concentration of impurities with a distribution of coupling constants can be embedded into an integrable correlated electron host [40] (or a Heisenberg chain [41]).

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